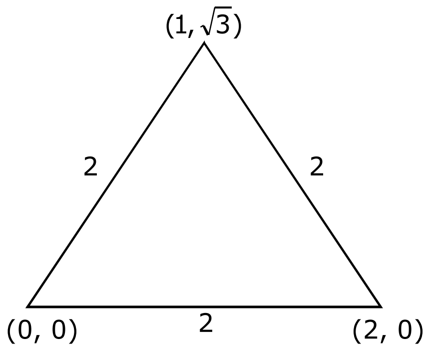
EXERCISE – V HINTS & SOLUTIONS

Sol.1 (a) Sides are 2

Δ is equlateral

$\Rightarrow$ Incentre is centroid

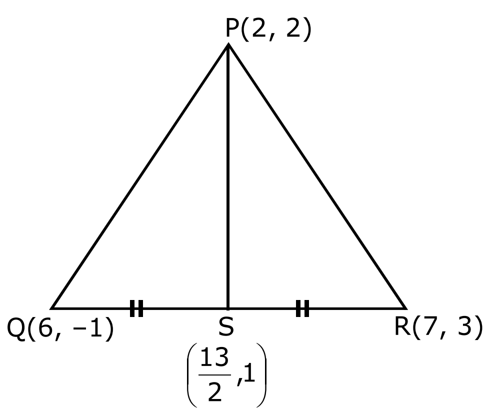


Incentre $\left(\frac{0+2+1}{3}, \frac{0+0+\sqrt{3}}{3}\right)$

$\Rightarrow \quad \left(1, \frac{1}{\sqrt{3}}\right)$

(b) S $\left(1, \frac{1}{\sqrt{3}}\right)$

$m_{PS} = \frac{2-1}{2-\frac{13}{2}} = \frac{1}{\frac{-9}{2}} = \frac{2}{-9}$



passes through (1, –1)

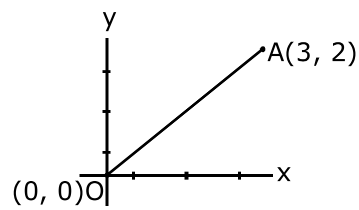
$y + 1 = \frac{2}{-9}(x - 1)$

$2x + 9y + 7 = 0$

(c) distance $d(P, Q) = |x_1 - x_2| + |y_2 - y_2|$,

$P(x, y)$, $Q(x_2, y_2)$

O (0, 0), A(3, 2), & Let P(h, k)



P lies in Ist Quadrant

d(P, O) = d (P,A)

$|h| + |k| = |h - 3| + |k - 2|$

$h_1k > 0$

$h + k = |h - 3| + |k - 2|$

$h_1k > 0$

$h + k = |h - 3| + |k - 2|$

Case-I

$0 < h < 3$, & $0 < k < 2$

$h + k = 3 - h + 2 - k$

$\Rightarrow \quad 2h + 2k = 5$

$\Rightarrow \quad 2x + 2y = 5$

For $\quad 0 < x < 3$

$0 < y < 2$

accept

Case-II

$0 < h < 3$ & $k \geq 2$

$h + k = 3 - h + k - 2$

$\Rightarrow \quad 2h = 1$

$2x = 1$ accept for $y \geq 2$

Case-III

$h \geq 3$ & $0 < k < 2$

$h + k = h - 3 + 2 - k$

$\Rightarrow \quad 2k + 1 = 0$

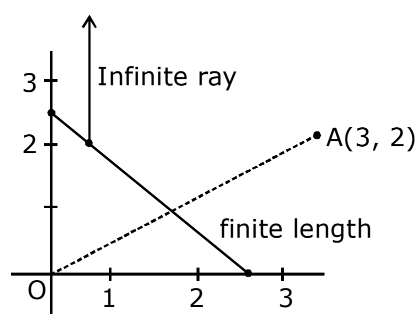
$2y + 1 = 0$

Case-IV

$h \geq 3$ & $k \geq 2$

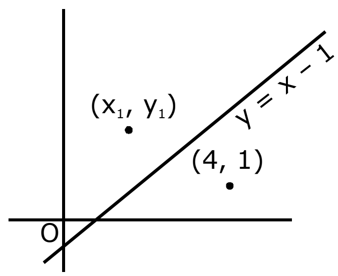
$h + k = h - 3 + k - 2$

N.S. reject



Set = {(x, y) : 2x + 2y = 5 if 0 < x < 3 and & 0 < y < 2 or 2x = 1 if y ≥ 2}

Sol.2 (i) Reflection about $y = x - 1$

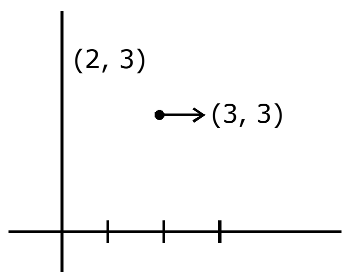


$$\frac{x_1 - 4}{1} = \frac{y_1 - 1}{-1} = \frac{-2(4-1-1)}{1^2 + (-1)^2}$$

$x_1 = 4 - 2,\ y_1 = +1 + 2$

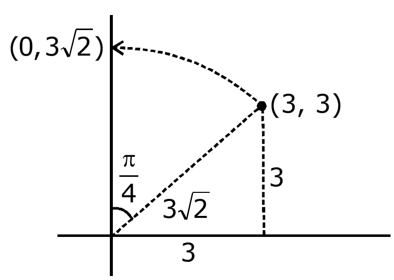
$x_1 = 2,\ y_1 = 3 \Rightarrow \quad (2, 3)$

(ii) One unit in + x-axis direction new position of point (3, 3)



(iii) Rotation through an angle $\pi/4$ about the origin in the anticlockwise direction

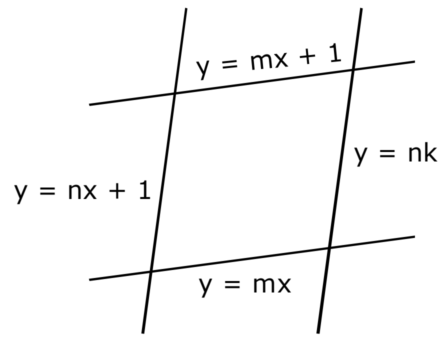
Finally coordinate of point



$(0,\ 3\sqrt{2})$

$(4, 1) \to (2, 3) \to (3, 3) \to (0, 3\sqrt{2})$

Solo.3 (a)



$$\text{Area} = \left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 m_2}\right|$$

$$= \left|\frac{(1-0)(1-0)}{m-n}\right|$$

$$= \frac{1}{|m-n|}$$

(b) $3x + 4y = 9,\ y = mx + 1$

$3x + 4mx + 4 = 9$

$x = \dfrac{5}{(3+4m)}$ 5 is divisible by $(3 + 4m)$

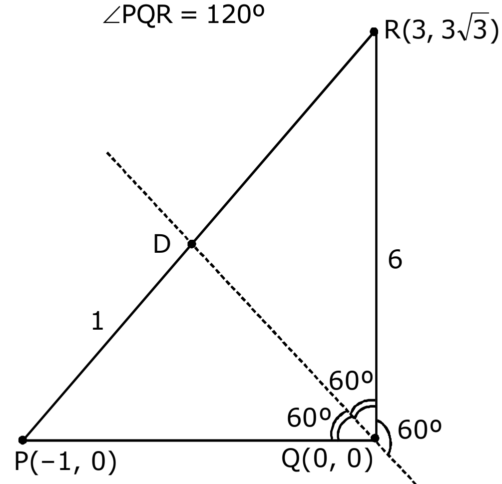
$4m + 3 = \pm 1$ or $4m + 3 = \pm 5$

$m = -\dfrac{1}{2}, -1$ or $m = \dfrac{1}{2}, -2$ Two values

Sol.4 **(a)** PQ lies on x-axis

$m_{QR} = \dfrac{3\sqrt{3}}{3} = \sqrt{3}$

$\angle PQR = 120^o$



inclination of required angle bisector is 120º.

Equation is $y = -\sqrt{3}\,x \Rightarrow \sqrt{3}\,x + y = 0$

Aliter :

PQ = 1

QR = 6 $D \equiv \left(\frac{-6+3}{6+1}, \frac{0+3\sqrt{3}}{6+1}\right) \equiv \left(\frac{-3}{7}, \frac{3\sqrt{3}}{7}\right)$

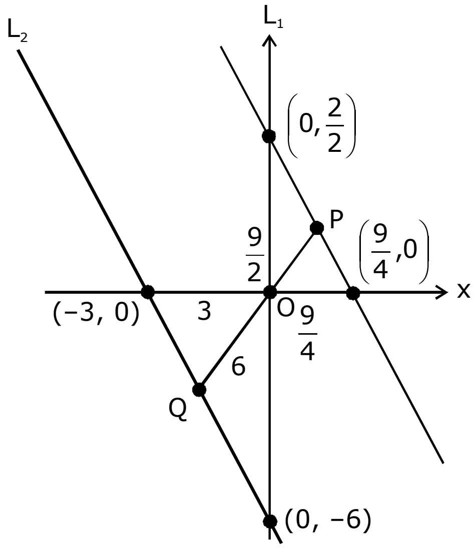
$\frac{PD}{RD} = \frac{1}{6}$

$m_{QD} = -\sqrt{3}$

$y - O = -\sqrt{3}\ (x - o) \Rightarrow \sqrt{3}x + y = 0$

(b) $L_1 : 4x + 2y = 9$

$L_2 : 2x + y + 6 = 0$



Perpendicular distance
Anyline passing through origin

$\frac{9}{4} : 3 \Rightarrow 3 : 4$

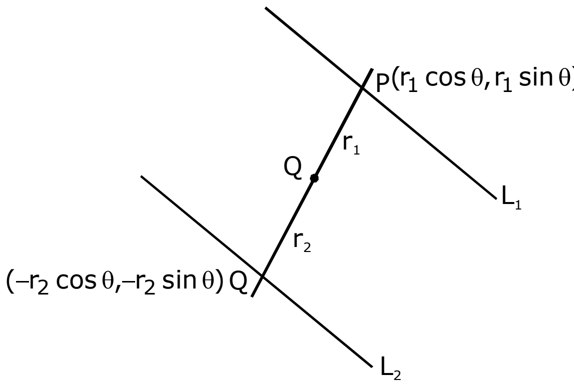
$\frac{9}{2} : 6 \Rightarrow 3 : 4$

$\frac{9}{2\sqrt{5}} : \frac{6}{\sqrt{5}} \Rightarrow 3 : 4$

Aliter :

Let a line passing through (0, 0) & slope $\tan\theta$

$\frac{x-0}{\cos\theta} = \frac{y-0}{\sin\theta} = r$



$4r_1\cos\theta + 2r_1\sin\theta = 9$

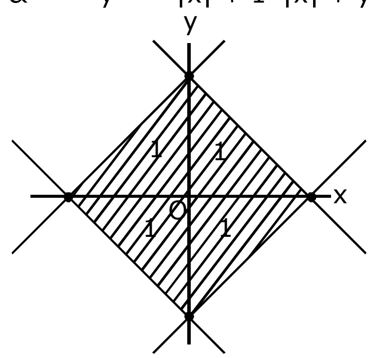
$2(r_2)\cos\theta + (-r_2)\sin\theta + 6 = 0$

$\frac{r_1}{(-r_2)} = \frac{\frac{9}{2}}{6} = \frac{3}{4}$

{In opposite sides r_2 is negative}

(c) $y = |x| - 1$ $\quad |x| - y = 1$

& $\quad y = -|x| + 1 \quad |x| + y = 1$



Area = $4\left(\frac{1}{2}.1.1\right)$ = 2 sq. units

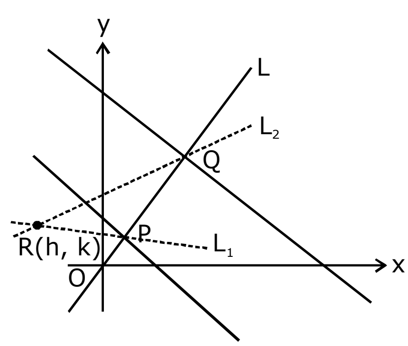
(d) $x + y = 1$

$x + y = 3$

L_1 is || to $2x - y = 5$

L_2 is || to $3x + y = 5$

Let line L = y = mx



$\therefore\ P\left(\frac{1}{1+m}, \frac{m}{1+m}\right)$ & $Q\left(\frac{3}{1+m}, \frac{3m}{1+m}\right)$

$\therefore\quad L_1 : 2x - y = \frac{2-m}{1+m}$

$L_2 : 3x + y = \frac{9+3m}{1+m}$

Intersection point R $h = \frac{11+2m}{5(1+m)}$

& $\quad k = \frac{12+9m}{5(1+m)}$

$\Rightarrow\quad m = \frac{11-5h}{5h-2} = \frac{12-5k}{5k-9}$

$\Rightarrow\quad 55k - 25hk - 99 + 45h$

$= 60h - 25hk - 24 + 10k$

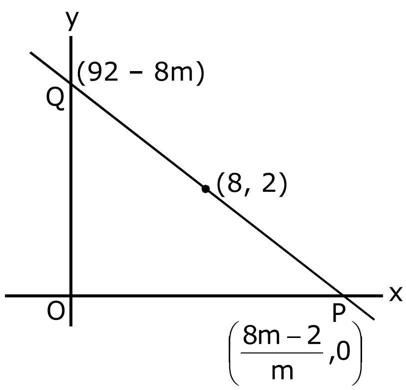
$\Rightarrow\quad 15h - 45k + 75 = 0$

$\Rightarrow\quad x - 3y + 5 = 0$

(e) Let line is

$(y - 2) = m(x - 8)$

$mx - y = 8m - 2$



$y = OP + OQ = \frac{(8m-2)}{m} + (2 - 8m)$

$y = (8m - 2)\left[\frac{1}{m-1}\right] = \frac{(8m-2)(1-m)}{m}$

$\Rightarrow\ my = -8m^2 + 10m - 2$

$\Rightarrow\ 8m^2 + m(y - 10) + 2 = 0$

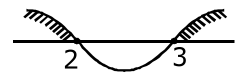
m should be negative & poroduct of roots > 0

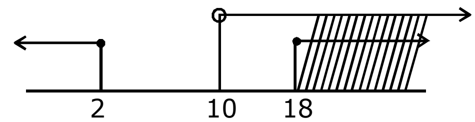
$\Rightarrow\quad$ sum of roots < 0

$\frac{-(y-10)}{8} < 0 \Rightarrow \quad y > 10$

& $b^2 - 4ac \geq 0$

$(y - 10)^2 - (8)^2 \geq 0 \Rightarrow (y - 2)(y - 18) \geq 0$





$y \in [18, \infty)$

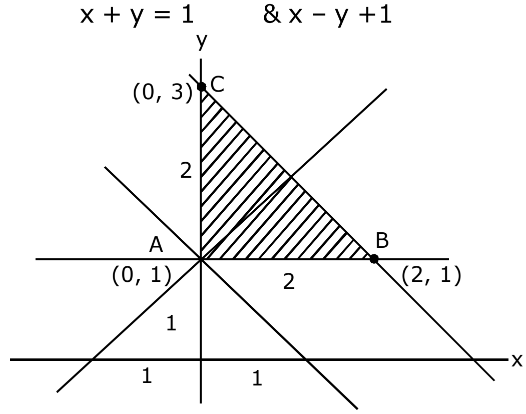
$y_{min} = 18$

Sol.5 $x^2 - y^2 + 2y - 1 = 0$

$x^2 (y - 1)^2 = 0$

$(x + y - 1)(x - y + 1) = 0$

$x + y = 1 \qquad \& \ x - y + 1$



angle bisector are

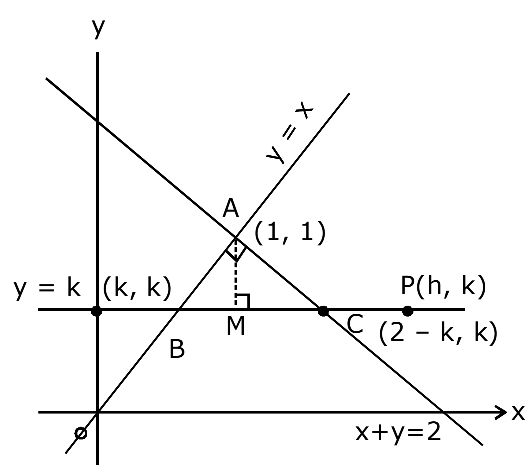
$y = 1$ & $x = 0$

A (0, 1), B(2, 1), C(0, 3)

area $\Delta ABC = \frac{1}{2}.\ 2.\ 2 = 2$ sq. units

Sol.6 $BC = 2 - k - k = 2 - 2k = 2(1 - k)$

$AB = (1 - k)$



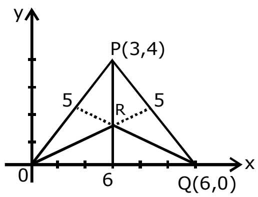
$$\Delta ABC = \left|\frac{1}{2}2(1-k)(1-k)\right| = 4h^2$$

$(1 - k)^2 = (2h)^2$
$1 - k = \pm 2h$
$y = \pm 2x + 1$

Sol.7 **(a)** centroid is that point s.t.
$\Delta OPR = \Delta OQR = \Delta PQR$

$$R\left(\frac{0+3+6}{3}, \frac{0+4+0}{3}\right)$$

$$R \equiv \left(3, \frac{4}{3}\right)$$



(b) $L_1 : y - x = 0$
$L_2 : 2x + y = 0$
$L_3 : y + 2 = 0$
Statement II is false $\Rightarrow$ (C)

S-I $OP = 2\sqrt{2}$, $OQ = \sqrt{5}$

$$\frac{PR}{RQ} = \frac{2\sqrt{2}}{\sqrt{5}} = \frac{OP}{OQ}$$

Sol.8 $L_1 : x + 3y - 5 = 0$
$L_2 : 3x - ky - 1 = 0$
$L_3 : 5x + 2y - 12 = 0$

(A) $\begin{vmatrix} 1 & 3 & -5 \\ 3 & -k & -1 \\ 5 & 2 & -12 \end{vmatrix} = 0$

$\Rightarrow -3(-36+10) - k(-12+25) + 1(2-15) = 0$
$\Rightarrow 78 - 13k - 13 = 0$
$\Rightarrow 13k = 65 \quad \Rightarrow \quad k = 5$

(B) $\frac{-1}{3} = \frac{3}{k} \quad \Rightarrow \quad k = -9$

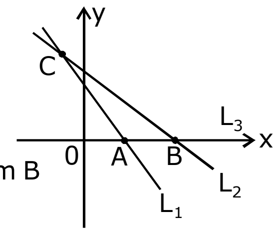
or $\frac{3}{k} = \frac{-5}{2} \Rightarrow k = -\frac{6}{5}$

(C) $k \neq 0, k \neq -9, k \neq -\frac{6}{5}$

$\Rightarrow \quad k = \frac{5}{6}$

(D) $k = 5,\ k = -9, k = -\frac{6}{5}$

Sol.9 $L_1 : (1 + p)x - py + p(1 + p) = 0$
$L_2 : (1 + q)x - qy + q(1 + q) = 0$
$L_3 : y = 0$
$A(-p, 0), B(-q, 0)$
Altitude from A



$$y = -\frac{q}{1+q}(x + p)$$

& Altitude from B

$$y = -\frac{p}{1+p}(x + q)$$

Solving these Altitudes

$$x\left(\frac{q}{1+q} - \frac{p}{1+p}\right) = pq\left(\frac{1}{1+p} - \frac{1}{1+q}\right)$$

$x(q - p) = pq(q - p)$

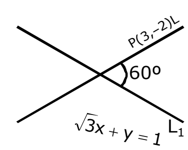
$x = pq$ & $\quad y = \frac{-q}{1+q}(pq + p)$

$$= -\frac{-q}{(1+q)}p(1 + q)$$

$q = -pq$
Orthocentre is $(h, k) \equiv (pq, -pq)$
$h = pq$ & $\quad k = -pq$
$h - pq = 0$
$h + k = 0$
$x + y = 0$ which is straight line

Sol.10 B

$M_{L_1} = -\sqrt{3}$
Let slope of L = m



$$\frac{m+\sqrt{3}}{1-m.\sqrt{3}} = \pm \sqrt{3}$$

on solving $m = \sqrt{3}, 0$

equation of line : $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$